Share nicely

The Strategy Unit.

NHS
Midlands and Lancashire
Commissioning Support Unit

Cooperation

Cooperation can often lead to outputs which are

“greater than the sum of the parts”

More savings/gains

How should we then share the gains?

Fairly

Participation in a coalition is voluntary

If participants collaborate, then they'll have to share their accumulated gains.

A fair approach to sharing should provide no incentive for players to break away from the coalition and proceed independently

No participant should feel disappointed – else may leave the coalition.

Three approaches

- Equal division – egalitarian
- Proportional division – eg by volume of patients
- Negotiation – cooperation and compromise

May appear unfair to one or more players

Shapley came up with a way of dividing the accumulated gains in such a case, called the <u>Shapley value.</u>

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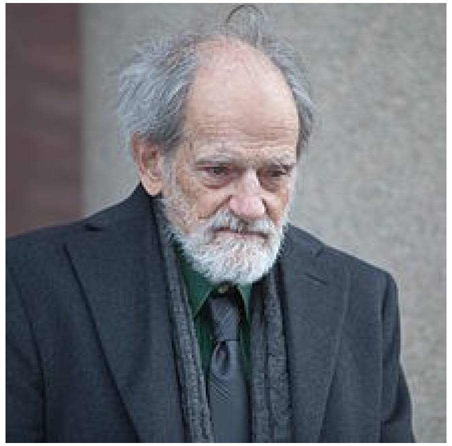
The Shapley Value

The Shapley value is a solution concept in cooperative game theory. It was named in honour of Lloyd Shapley, who introduced it in 1951 and won the Nobel Memorial Prize in Economic Sciences for it in 2012.

$$\varphi_i(v) = \frac{1}{n} \sum_{S \subseteq N \setminus \{i\}} \binom{n-1}{|S|}^{-1} \left(v(S \cup \{i\}) - v(S)\right)$$

which can be interpreted as

$$\varphi_i(v) = \frac{1}{\text{number of players}} \sum_{\text{coalitions including } i} \frac{\text{marginal contribution of } i \text{ to coalition}}{\text{number of coalitions excluding } i \text{ of this size}}$$



The Shapley value is a solution concept used in game theory. The Shapley value is mainly applicable to the following situation: the contribution of each actor is not equal, but each participant cooperates with each other to obtain profit or return. The efficiency of the resource allocation and combination of the two distribution methods are more reasonable and fair, and it also reflects the process of mutual game among the league members.[4] However, the benefit distribution plan of the Shapley value method has not considered the risk sharing factors of organization members, which essentially implies the assumption of equal risk sharing.

Airport Problem

In mathematics and especially game theory, the **airport problem** is a type of fair division problem in which it is decided how to distribute the cost of an airport runway among different players who need runways of different lengths. The problem was introduced by S. C. Littlechild and G. Owen in 1973.[1] The authors note that the resulting set of landing charges is the Shapley value for an appropriately defined game.

Sharing nicely

Player 1 - £1 : Player 2 - £2 Player 1 + Player 2 = £4

How should they (fairly) share the £4 profit

The total £ (when each player is alone) = £3

The extra marginal gain from working together = +£1

Split the **marginal gain** +£1 **equally** –

Player 1 gets: £1 + £0.50p = £1.50

Player 2 gets: £2 + £0.50p = £2.50

Sharing nicely

Three friends want to share a taxi home: A (£6) ----->B (£12) -------------->C (£42)

The taxi firm agrees to a fare of £42 total based on longest journey to C

It is now up to the passengers to divide the fare between them

	A - > £6 :	B -> £12 :	C -> £42	
Coalitions	**A**	**B**	**C**	
ABC	6	6	(42-6-6)=30	
ACB	6	0	(42-6)=36	
BAC	0	12	(42-12)=30	
BCA	0	12	(42-12)=30	
CAB	0	0	42	
CBA	0	0	42	
Fare	12/6	30/6	210/6	
	£2	£5	£35	Total sum = £42

CT SCANNER

Parallel Simplification: The three hospital networks will purchase and share use of a new Aquilion One Vision CT scanner. Hospital X_1 has modest general-imaging requirements and wants a 16-slice scanner ($400,000) to meet their needs. Hospital X_2 has higher volume requirements and also wishes to offer vascular imaging services which need better quality images, so they require a 128-slice system ($1,000,000). Finally, hospital X_3 wishes to setup a cutting-edge imaging facility capable of high volume cardio studies; consequently, they require a 640-slice scanner ($2,500,000). Given the expected usage volume, all three hospitals could share a single 640-slice machine with this cost allocation.

Resource Cost		Player Requirement			Cost to "go it alone"
16-slice:	$ 400,000	S(1):	16-slice	$400,000	X_1 = 400,000
128-slice:	$ 1,000,000	S(2):	128-slice	$1,000,000	X_2 = 1,000,000
640-slice:	$ 2,500,000	S(3):	640-slice	$2,500,000	X_3 = 2,500,000
					==========
					$ 3,900,000

Shapley Sharing Solution

$X_1 = ⅓ (400,000) = 133,333.33$

$X_2 = ⅓(400,000) + ½(1,000,000 – 400,000) = 433,333.33$

$X_3 = ⅓(400,000) + ½(1,000,000 – 400,000) + (2,5000,000 – 1,000,000) = 1,933,333.34$

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$ 2,500,000.00

Lightfoot, Jay M. (2019) "A Game-Theoretic Approach to Share the Costs of Cooperating Healthcare Networks," Journal of International Technology and Information Management: Vol. 27: Iss. 3, Article 2. DOI: https://doi.org/10.58729/1941-6679.1381

Shapley value

- Fair share
- Assumes constant risk

Reflections

Share your insights…

Any key insights...

So what…

(any scope for application)

One wish…